17 *Years*

Previous Solved Papers

GATE 2024

Production & Industrial Engineering

- Fully solved with explanations
- Topicwise presentation
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GATE - 2024
Production & Industrial Engineering
Topicwise Previous GATE Solved Papers (2007-2023)

Editions

1st Edition : 2020
2nd Edition : 2021
3rd Edition : 2022

4th Edition : 2023

Preface

B. Singh (Ex. IES)

Over the period of time the GATE examination has become more challenging due to increasing number of candidates. Though every candidate has ability to succeed but competitive environment, in-depth knowledge, quality guidance and good source of study is required to achieve high level goals.

The new edition of **GATE 2024 Solved Papers : Production & Industrial Engineering** has been fully revised, updated and edited. The whole book has been divided into topicwise sections.

At the beginning of each subject, analysis of previous papers are given to improve the understanding of subject.

I have true desire to serve student community by way of providing good source of study and quality guidance. I hope this book will be proved an important tool to succeed in GATE examination. Any suggestions from the readers for the improvement of this book are most welcome.

B. Singh (Ex. IES)
Chairman and Managing Director
MADE EASY Group

GATE-2024

Production & Industrial Engg.

CONTENTS

General Engineering

UNIT

II

CONTENTS

General Engineering

Syllabus

Engineering Materials: Structure, physical and mechanical properties, and applications of common engineering materials (metals and alloys, semiconductors, ceramics, polymers, and composites – metal, polymer and ceramic based); Iron-carbon equilibrium phase diagram; Heat treatment of metals and alloys and its influence on mechanical properties; Stress-strain behavior of metals and alloys.

Applied Mechanics: Engineering mechanics – equivalent force systems, free body concepts, equations of equilibrium; Trusses; Strength of materials – stress, strain and their relationship; Failure theories; Mohr's circle (stress); Deflection of beams, bending and shear stresses; Euler's theory of columns; Thick and thin cylinders; Torsion.

Theory of Machines and Design: Analysis of planar mechanisms, cams and followers; Governors and fly wheels; Design of bolted, riveted and welded joints; Interference/shrink fit joints; Friction and lubrication; Design of shafts, keys, couplings, spur gears, belt drives, brakes and clutches; Pressure vessels.

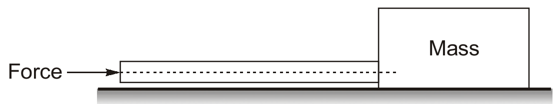
Thermal and Fluids Engineering: Fluid mechanics – fluid statics, Bernoulli's equation, flow through pipes, laminar and turbulent flows, equations of continuity and momentum, capillary action; Dimensional analysis; Thermodynamics – zeroth, first and second laws of thermodynamics, thermodynamic systems and processes, calculation of work and heat for systems and control volumes; Air standard cycles; Heat transfer – basic applications of conduction, convection and radiation.

2 Applied Mechanics

2.1 A component made of a material with a modulus of elasticity of 200 MPa and modulus of rigidity of 80 MPa experiences an axial strain of 1000. The lateral strain experienced by the component within the elastic limit is

(a) 250 (b) 400
(c) 500 (d) 800 **[2007 : 1 M]**

2.2 Figure below shows a mass of 300 kg being pushed using a cylindrical rod made of a material having E = 22 MPa and of 2 m length and 0.1 m in diameter. In order to avoid the failure of the rod due to elastic instability, the maximum value of the coefficient of Coulomb friction permissible between the mass and the floor is



(a) 0.22 (b) 0.36
(c) 0.65 (d) 0.75 **[2007 : 2 M]**

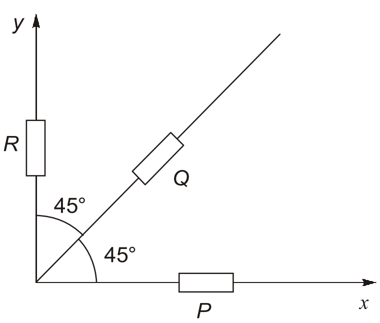
2.3 The state of stress at a point in a body under plane stale of stress condition is given by $\begin{bmatrix} 60 & 0 \\ 0 & 20 \end{bmatrix}$ MPa. The maximum shear stress (in MPa) is

(a) 0 (b) 20
(c) 30 (d) 40 **[2008 : 1 M]**

2.4 A shaft of diameter 10 mm transmits 100 W of power at an angular speed of $\frac{800}{\pi}$ rad/s. The maximum shear stress (in MPa) developed in the shaft is

(a) 2 (b) 4
(c) 8 (d) 16 **[2008 : 2 M]**

2.5 A strain rosette, as shown in the figure, has three strain gauges P, Q and R.



If the values of strain indicated in the three strain gauges are

$\varepsilon_P = 100 \times 10^{-6}$
$\varepsilon_Q = 150 \times 10^{-6}$ and
$\varepsilon_R = 200 \times 10^{-6}$,

the largest principal strain is

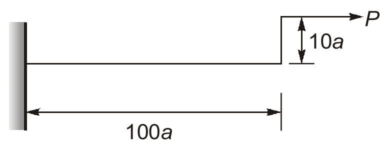
(a) 200×10^{-6} (b) 250×10^{-6}
(c) 300×10^{-6} (d) 350×10^{-6} **[2008 : 2 M]**

2.6 A cantilever beam XY of length 2 m and cross-sectional dimensions 25 mm × 25 mm is fixed at X and is subjected to a moment of 100 N-m and an unknown force P at the free end Y as shown in the figure. The Young's modulus of the material of the beam is 200 GPa.

If the deflection of the free end Y is zero, then the value of P (in N) is

(a) 67 (b) 75
(c) 133 (d) 150 **[2008 : 2 M]**

2.7 A frame of square cross-section of ($a \times a$) is as shown in the figure. The stress near the fixed end on the upper side of the frame is



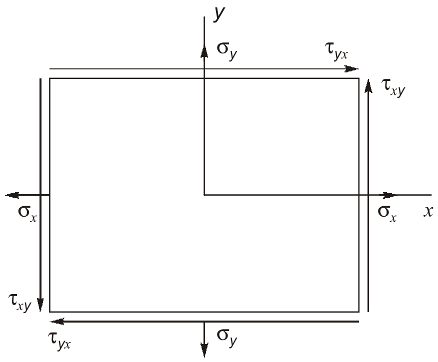
(a) $\frac{58P}{a^2}$ (b) $\frac{59P}{a^2}$
(c) $\frac{60P}{a^2}$ (d) $\frac{61P}{a^2}$ **[2008 : 2 M]**

2.8 A steel wire of diameter 2 mm is wound on a rigid drum of diameter 2 m. If the Young's modulus of the steel is 200 GPa, the maximum stress (in MPa) in the steel wire is

(a) 50 (b) 100
(c) 200 (d) 400 **[2008 : 2 M]**

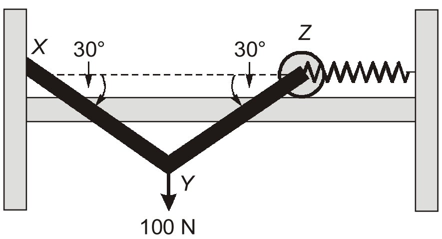
2.9 A biaxial stress element is subjected to tensile and shear stresses as shown in the figure. If σ_x = 40 MPa, σ_y = 20 MPa and $\tau_{xy} = \tau_{yx}$ = 15 MPa.

The principal normal stresses (in MPa) are:



(a) 5 and 55 (b) 10 and 30
(c) 12 and 48 (d) 20 and 40 [2009 : 2 M]

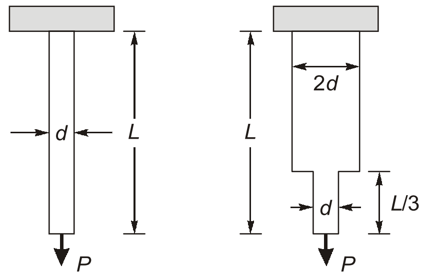
2.10 A rigid massless link *YZ* of length 100 mm is connected at one end to another massless link *XY* of the same length by means of a frictionless hinge at Y and at the other end to a fictionless roller, as shown in the following figure. The link *XY* is connected to the wall by means of a frictionless hinge at point *X*. The roller is connected to a massless linear spring with a spring constant 10 kN/m. A point force of 100 N is applied at point *Y* as shown in the figure. At equilibrium, each of the links *XY* and *YZ* makes an angle $\theta = 30°$ with the horizontal. Under this situation, the stretch of the spring (in mm) is



(a) $\frac{5}{3}\sqrt{3}$ (b) $\frac{5}{2}\sqrt{3}$
(c) $5\sqrt{3}$ (d) $10\sqrt{3}$ [2010 : 2 M]

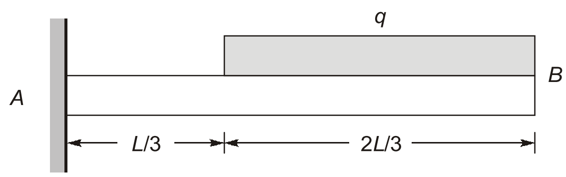
2.11 A circular steel shaft is under elastic deformation due to torsion. The relationship between modulus of elasticity (*E*) and shear modulus of elasticity (*G*), taking ν/m as Poisson's ratio, is
(a) $G = 2E(1 + \nu)$ (b) $E = 2G(1 + \nu)$
(c) $G = \dfrac{2E}{(1+\nu)}$ (d) $E = \dfrac{2G}{(1+\nu)}$
[2011 : 1 M]

2.12 Two circular steel bars having same length *L* are subjected to equal load *P*. The first bar has diameter *d* over its entire length, while the second bar has diameter 2*d* over two-thirds of its length as shown in the figure. Assuming linear elastic behaviour, the ratio of strain energy of the first bar to that of the second bar is



(a) $\frac{1}{2}$ (b) 4
(c) $\frac{1}{4}$ (d) 2 [2011 : 1 M]

2.13 A cantilever beam *AB* of length *L*, rigidly fixed at end *A*, is uniformly loaded with intensity *q* (downwards) over two-thirds of its length from the free end *B* as shown in the figure. The modulus of elasticity is *E* and the moment of inertia about the horizontal axis is *I*. The angle of rotation at the free end under the applied load is



(a) $\dfrac{7qL^3}{48EI}$ (b) $\dfrac{13qL^3}{72EI}$
(c) $\dfrac{11qL^3}{60EI}$ (d) $\dfrac{qL^3}{24EI}$ [2011 : 2 M]

2.14 A short column of length *L* having cross-sectional area of 50 mm by 100 mm is pinned at the ends. The proportional limit of the column is 250 MPa and modulus of elasticity is 200 GPa. The minimum length of the column (in m) at which it will buckle elastically is
(a) 5.25 (b) 2.25
(c) 1.65 (d) 1.15 [2011 : 2 M]

2.15 For a long slender column of uniform cross section, the ratio of critical buckling load for the case with both ends clamped to the case with both ends hinged is
(a) 1 (b) 2
(c) 4 (d) 8 [2012 : 1 M]

2.16 A cantilever beam of length *L* is subjected to a moment *M* at the free end. The moment of inertia of the beam cross section about the neutral axis is *I* and the Young's modulus is *E*. The magnitude of the maximum deflection is

(a) $\frac{ML^2}{2EI}$ (b) $\frac{ML^2}{EI}$

(c) $\frac{2ML^2}{EI}$ (d) $\frac{4ML^2}{EI}$ [2012 : 1 M]

2.17 The state of stress at a point under plane stress condition is

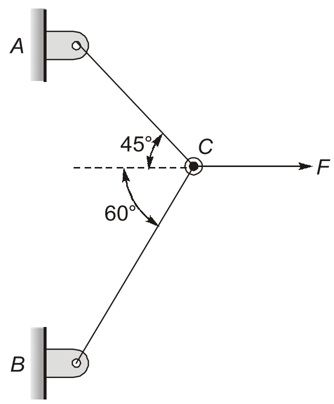
σ_{xx} = 40 MPa, σ_{yy} = 100 MPa and τ_{xy} = 40 MPa. The radius of the Mohr's circle representing the given state of stress in MPa is

(a) 40 (b) 50

(c) 60 (d) 100 [2012 : 2 M]

Common Data for Questions 2.18 and 2.19 :

Two steel truss members, *AC* and *BC*, each having cross sectional area of 100 mm^2, are subjected to a horizontal force *F* as shown in figure. All the joints are hinged.



2.18 The maximum force *F* in kN that can be applied at *C* such that the axial stress in any of the truss members DOES NOT exceed 100 MPa is

(a) 8.17 (b) 11.15

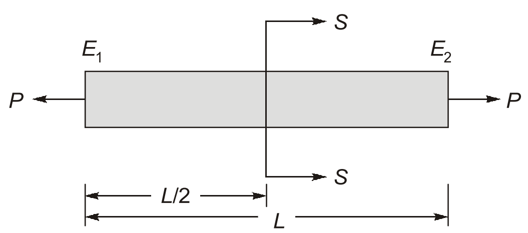
(c) 14.14 (d) 22.30 [2012 : 2 M]

2.19 If *F* = 1 kN, the magnitude of the vertical reaction force developed at the point *B* in kN is

(a) 0.63 (b) 0.32

(c) 1.26 (d) 1.46 [2012 : 2 M]

2.20 A rod of length *L* having uniform cross-sectional area *A* is subjected to a tensile force *P* as shown in the figure below. If the Young's modulus of the material varies linearly from E_1 to E_2 along the length of the rod, the normal stress developed at the section-SS is



(a) $\frac{P}{A}$ (b) $\frac{P(E_1 - E_2)}{A(E_1 + E_2)}$

(c) $\frac{PE_2}{AE_1}$ (d) $\frac{PE_1}{AE_2}$ [2013 : 1 M]

2.21 A simply supported beam of length *L* is subjected to a varying distributed load sin ($3\pi\ x/L$) Nm^{-1}, where the distance is measured from the left support. The magnitude of the vertical reaction force in *N* at the left support is

(a) zero (b) $L/3\pi$

(c) L/π (d) $2L/\pi$ [2013 : 2 M]

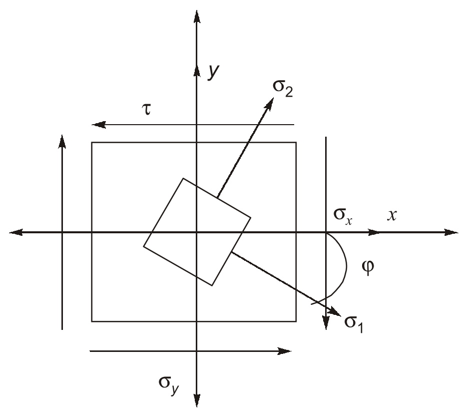
2.22 Relationship between Young's modulus (*E*), Shear modulus (*G*), and Poisson's ratio (μ), for a material obeying the Hooke's Law, is

(a) $E = \frac{G}{(2+\mu)}$ (b) $E = \frac{2G}{(1+\mu)}$

(c) $E = G(1 + \mu)$ (d) $E = 2G(1 + \mu)$

[2014 : 1 M]

2.23 An element, shown below, is subjected to stresses: σ_x = 5 kN/mm^2, σ_y = 3 kN/mm^2 and τ = 1 kN/mm^2. The magnitudes and direction of principal stresses σ_1, σ_2 (in kN/mm^2) and φ (in degrees) are

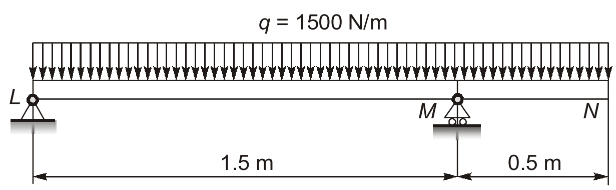


(a) 5.41, 2.58, –22.5 (b) 5.41, 2.58, –45

(c) 5.0, 3.0, –22.5 (d) 4.0, 4.0, –22.5

[2014 : 2 M]

2.24 A uniformly distributed load (*q*) of 1500 N/m is applied on a simply supported beam LMN with an overhang of 0.5 m. Which one of the following statements is FALSE?



(a) Reaction forces at L and M are 1 kN and 2 kN
(b) Bending moment is zero at the points L, N and at a point in between L and M
(c) The bending moment is zero at points L and N only
(d) The shear force is zero at points L, N and at a point in between L and M [2014 : 2 M]

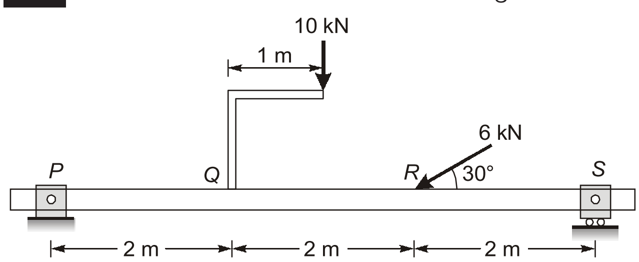
2.25 A hard ceramic marble, having density (ρ) of 3000 kg/m^3 and diameter (d) of 0.025 m, is dropped accidentally from a static weather balloon at a height of 1 km above the roof of a greenhouse. The flow stress of roof material (σ) is 2.5 GPa. The marble hits and creates an indentation on the roof. Assume that the principle of creation of indentation is the same as that in case of abrasive jet machining (AJM). The acceleration due to gravity (g) is 10 m/s^2. If V is the velocity, in m/s, of the marble at the time it hits the greenhouse, the indentation depth ($\delta = 1000 \times \sqrt{\frac{\rho}{6\sigma}} \times d \times V$), in mm is _______. [2014 : 2 M]

2.26 The true stress at fracture of a tensile tested specimen, having an initial diameter of 13 mm, is 700 MPa. If the diameter of specimen at fracture is 10 mm, then the engineering stress, in MPa, at fracture is __________. [2015 : 1 M]

2.27 If the principal stress values are 120 MPa, –50 MPa and 10 MPa in a given state of stress, then maximum shear stress in the material, in MPa, is __________. [2015 : 1 M]

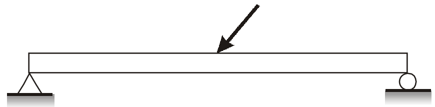
2.28 A metallic bar of uniform cross-section with specific weight of 100 kN/m^3 is hung vertically down. The length and Young's modulus of the bar are 100 m and 200 GPa, respectively. The elongation of the bar, in mm, due to its own weight is __________. [2015 : 2 M]

2.29 A beam is loaded as shown in the figure.



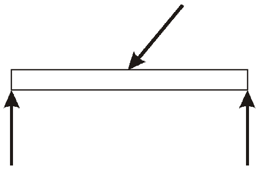
The bending moment, in Nm, at point R is __________. [2015 : 2 M]

2.30 A beam is subjected to an inclined concentrated load as shown in the figure below. Neglect the weight of the beam.

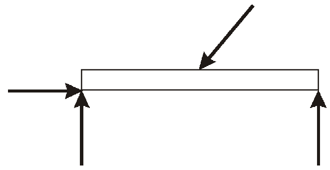


The correct Free Body Diagram of the beam is

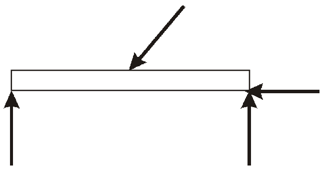
(a)



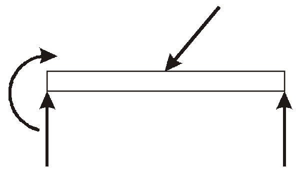
(b)



(c)

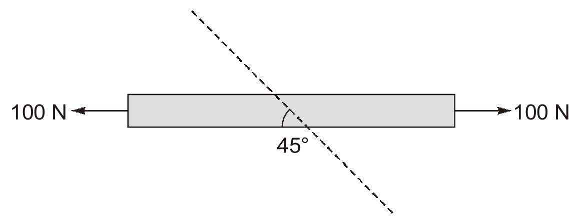


(d)



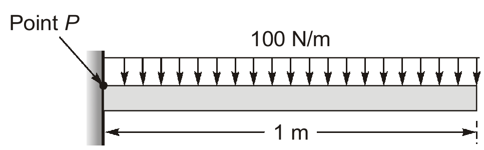
[2016 : 1 M]

2.31 A bar of rectangular cross-sectional area of 50 mm^2 is pulled from both the sides by equal forces of 100 N as shown in the figure below. The shear stress (in MPa) along the plane making an angle 45° with the axis, shown as a dashed line in the figure, is ____________.



[2016 : 2 M]

2.32 A 1 m × 10 mm × 10 mm cantilever beam is subjected to a uniformly distributed load per unit length of 100 N/m as shown in the figure below. The normal stress (in MPa) due to bending at point P is __________.

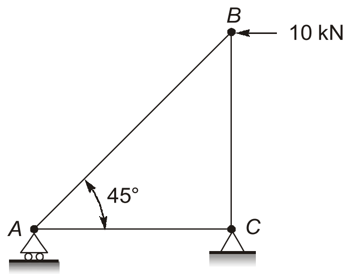


[2016 : 2 M]

2.56 A thick-cylinder has inner diameter of 20 mm and outer diameter of 40 mm. It is subjected to an internal pressure of 100 MPa. Follow the convention of taking tensile stress as positive and compressive stress as negative. The sum of radial and hoop stresses (in MPa) at a radius of 15 mm is_______. [Round off to two decimal places]

[2022 : 1 M]

2.57 In the three-member truss shown in the figure, $AC = BC$. An external force of 10 kN is applied at B, parallel to AC. The force in the member BC is



(a) 10 kN (compressive)
(b) 7.07 kN (tensile)
(c) 10 kN (tensile)
(d) zero

[2022 : 2 M]

2.58 A thin cylinder has length L, diameter d, and thickness t. It is made of a material with modulus of elasticity E and Poisson's ratio μ. When the cylinder is subjected to an internal pressure P, the change in length is

(a) $\frac{PdL}{2tE}\left(\frac{1}{2}-\mu\right)$ (b) $\frac{PdL}{2tE}(2-\mu)$

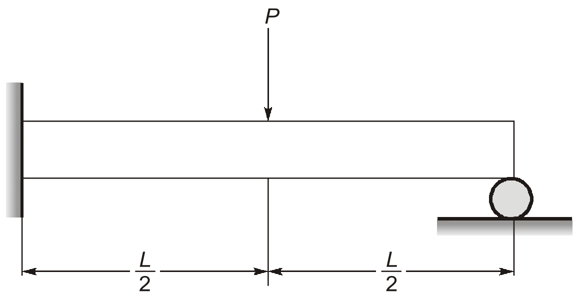
(c) $\frac{PdL}{2tE}(1-2\mu)$ (d) $\frac{PdL}{4tE}\left(\frac{1}{2}-\mu\right)$

[2023 : 1 M]

2.59 A solid circular disk of 0.025 m thickness is used as flywheel. The density of the disk material is 7800 kg/m^3 and the mass moment of inertia of the disk about its center is 4.36 kg-m^2. The radius, in m, of the disk is _____(round off to 2 decimal places).

[2023 : 1 M]

2.60 A massless beam is fixed at one end and supported on a roller at other end. A point force P is applied at the midpoint of the beam as shown in figure. The reaction at the roller support is



(a) $\frac{5P}{16}$ (b) $\frac{2P}{3}$

(c) $\frac{4P}{9}$ (d) $\frac{9P}{25}$

[2023 : 2 M]

■■■■

Answers Applied Mechanics

2.1 (a)	2.2 (b)	2.3 (b)	2.4 (a)	2.5 (a)	2.6 (b)	2.7 (d)
2.8 (c)	2.9 (c)	2.10 (d)	2.11 (b)	2.12 (d)	2.13 (b)	2.14 (d)
2.15 (c)	2.16 (a)	2.17 (b)	2.18 (b)	2.19 (a)	2.20 (a)	2.21 (b)
2.22 (d)	2.23 (a)	2.24 (c)	2.25 (1.5811)	2.26 (414.201)	2.27 (85)	2.28 (2.5)
2.29 (14)	2.30 (b)	2.31 (1)	2.32 (300)	2.33 (0.81)	2.34 (a)	2.35 (1.4)
2.36 (b)	2.37 (d)	2.38 (18.75)	2.39 (a)	2.40 (20,000)	2.41 (1600)	2.42 (0.9428)
2.43 (b)	2.44 (c)	2.45 (d)	2.46 (a)	2.47 (404.14)	2.48 (0.18)	2.49 (–5773.5)
2.50 (c)	2.51 (147.15)	2.52 (a)	2.53 (b)	2.54 (c)	2.55 (100)	2.56 (66.66)
2.57 (c)	2.58 (a)	2.59 (0.35)	2.60 (a)			

Explanations Applied Mechanics

2.1 (a)

Given : Modulus of elasticity, E = 200 MPa
Modulus of rigidity, C = 80 MPa
Axial strain = Longitudinal Strain = 1000
To find : Lateral strain
We know that Poisson's ration, μ

$$\mu = \frac{\text{Lateral strain}}{\text{Longitudinal strain}}$$

and Poisson's ratio can be calculated using

$$E = 2C(1+\mu)$$

On substituting values,

$$200 = 2 \times 80(1+\mu)$$
$$1.25 = 1 + \mu \Rightarrow \mu = 0.25$$
$$0.25 = \frac{\text{Lateral strain}}{1000}$$
$$\text{Lateral strain} = 250$$

2.2 (b)

Given :
Mass of block, M = 300 kg
E of rod, Young's modulus,
E = 22 MPa
Length of rod, L = 2 m
Diameter of rod, d = 0.1 m
To find : Maximum value of coefficient of friction, μ between block and ground to prevent failure of rod due to elastic instability.

2.3 (b)

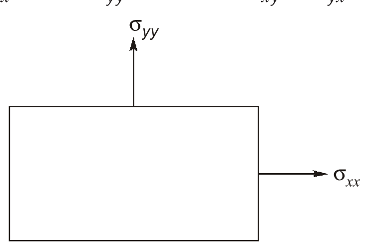
Given : Stress modification as $\begin{bmatrix} 60 & 0 \\ 0 & 20 \end{bmatrix}$ MPa

To find : Maximum shear stress

We know that the stress tenser is $\begin{bmatrix} \sigma_{xx} & \sigma_{xy} \\ \sigma_{yx} & \sigma_{yy} \end{bmatrix}$

On comparing with the given stress tensor, we found that

$$\sigma_{xx} = 60, \sigma_{yy} = 20 \text{ and } \sigma_{xy} = \sigma_{yx} = 0$$



So, this can be expressed as shown in figure. We know that maximum shear stress is given by radius of Mohr's circle which is given as

$$\tau_{max} = \frac{1}{2}\sqrt{(\sigma_y - \sigma_x)^2 + 4\sigma_{xy}^2}$$
$$= \frac{1}{2}\sqrt{(60-20)^2 - 0^2} = \frac{1}{2} \times 40 = 20 \text{ MPa}$$

2.4 (a)

Given: Diameter of shaft, d = 10 mm

Power to be transmitted, P = 100 W

Angular speed, $\omega = \frac{800}{\pi}$ rad/sec

To find maximum shear stress developed in the shaft.

We know that power, $P = T\omega$

T = Torque

$$100 = T \times \frac{800}{\pi}$$

$$\Rightarrow \quad T = \frac{100\pi}{800} \text{N/m}$$

We also know,

$$\frac{T}{J} = \frac{\tau}{R}$$

T = Torque
J = Polar Moment of inertia
τ = Shear stress developed
R = Radius of shaft

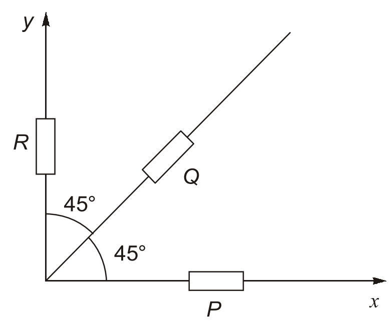
$$\frac{100\pi \times 1000 \times 32}{800 \times \pi \times (10)^4} = \frac{\tau}{10/2}$$

$$\frac{10 \times 1000 \times 32}{800 \times 1000} = \tau$$

$$\tau = 2 \text{ N/mm}^2 = 2 \text{ MPa}$$

2.5 (a)

Given : A strain rosette arrangement as shown.



Value of strains,

$$\in_P = 100 \times 10^{-6}$$
$$\in_Q = 150 \times 10^{-6}$$
and $\in_R = 200 \times 10^{-6}$

To find : Largest principal strain.

We know that for the given arrangement of strain rossette, the principal strains can be calculated as

$$\in_1 / \in_2 = \frac{1}{2}\cdot(\in_P + \in_R) \pm \frac{1}{2}\sqrt{(\in_P - \in_R)^2 + (2\cdot\in_Q - \in_P - \in_R)^2}$$

Here, we are required to find out maximum strain so we will use '+' sign.

$$\in_1 = \frac{1}{2}(\in_P + \in_R) + \frac{1}{2}\sqrt{(\in_P - \in_R)^2 + (2\cdot\in_Q - \in_P - \in_R)^2}$$

On substituting the values, we get

$$\in_1 = \frac{1}{2}(100 \times 10^{-6} + 200 \times 10^{-6}) + \frac{1}{2}\sqrt{(100\times10^{-6} - 200\times10^{-6})^2 + (2(1.50\times10^{-6}) - 100\times10^{-6} - 200\times10^{-6})^2}$$

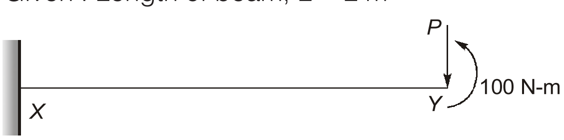
$$\in_1 = \frac{1}{2} \times 10^{-6} \times 300 + \frac{1}{2}\sqrt{(-100\times10^{-6})^2 + 0}$$

$$\in_1 = 150 \times 10^{-6} + \frac{1}{2} \times 100 \times 10^{-6} = 200 \times 10^{-6}$$

Maximum principal strain = 200×10^{-6}

2.6 (b)

Given : Length of beam, L = 2 m



Cross sectional dimension of beam

$$= 25 \times 25 \text{ mm}^2$$

Moment at free end Y,

$$M = 100 \text{ Nm}$$

Load at free end $Y = P$

Young's modulus,

$$E = 200 \text{ GPa}$$

To find : Value of P, so that deflection at Y is zero.

For deflection at Y to be zero, upward deflection of beam due moment should be equal to downward deflection of beam due to P.

Upward deflection due to moment is given as Y_M

$$Y_M = \frac{ML^2}{2EI} \quad ...(1)$$

Downward deflection due to point load, Y_P

$$Y_P = \frac{PL^3}{3EI} \quad ...(2)$$

On equating eqns., (1) and (2), we get

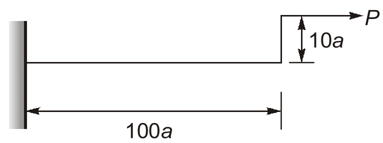
$$\frac{ML^2}{2EI} = \frac{PL^3}{3EI} \Rightarrow \frac{3}{2}\frac{M}{L} = P$$

$$\frac{3}{2} \times \frac{100 \text{ Nm}}{2 \text{ M}} = P$$

$$P = 75 \text{ N}$$

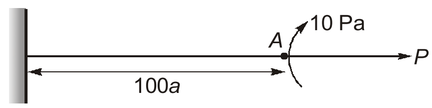
2.7 (d)

Given : Frame with square cross section ($a \times a$)



To find : Stress near the fixed end on the upper side of frame.

The loading on the frame can be simplified as a load P @ point A with a moment of 10 Pa in clockwise direction at A.



The stress at point A is direct stress due to load P and bending stress due to moment 10 Pa.

$$\text{Direct stress} = \frac{\text{Load}}{\text{Area}} = \frac{P}{a^2} = \frac{P}{a^2}$$

Bending stress is calculated using flexural formula

$$\frac{M}{I} = \frac{f}{y}$$

where M = Bending moment

I = Moment of inertia

f = Bending stress

and y = Distance of most distant fibre from neutral axis

$$M = 10 \text{ Pa}, I = \frac{a^4}{12}, y = \frac{a}{2}$$

$$\frac{10 \text{ Pa}}{a^4/12} = \frac{f}{a/2}$$

$$f = \frac{60P}{a^2}$$

Total stress = Direct stress + Bending stress

$$= \frac{P}{a^2} + \frac{60P}{a^2} = \frac{61P}{a^2}$$

2.8 **(c)**

Given: Diameter of wire,

$$d = 2 \text{ mm}$$

Diameter of rigid drum on which wire is wound

$$= 2 \text{ m}$$

Young's modulus of steel,

$$E = 200 \text{ GPa}$$

To find : Maximum stress in steel wire.

We know that maximum stress can be found using flexural formula,

$$\frac{M}{I} = \frac{f}{y} = \frac{E}{R}$$

using $\frac{f}{y} = \frac{E}{R}$

where, f = Bending stress

E = Young's modulus

R = Radius in which wire is wound

y = Distance of most distant fibre in wire from neutral axis of wire

Here, y = 1 mm, R = 1 m,

$E = 200 \times 1000$ MPa

On substituting these values, we get

$$\frac{f}{1 \text{ mm}} = \frac{200 \times 1000 \text{ N/mm}^2}{1000 \text{ mm}}$$

$$f = 200 \text{ N/mm}^2 = 200 \text{ MPa}$$

2.9 **(c)**

Given : Stress in x direction, σ_x = 40 MPa

Stress in y-direction, σ_y = 20 MPa

Shear stress, $\tau_{xy} = \tau_{yx}$ = 15 MPa

To find : Principal normal stresses

We know that principal stresses are given as

$$\sigma_{P_1}, \sigma_{P_2} = \frac{\sigma_x + \sigma_y}{2} \pm \frac{1}{2}\sqrt{(\sigma_y - \sigma_x)^2 + 4\tau_{xy}^2}$$

$$\sigma_{P_1}, \sigma_{P_2} = \frac{40 + 20}{2} \pm \frac{1}{2}\sqrt{(20 - 40)^2 + 4 \times 15^2}$$

$$= 30 \pm 18.027$$

So, σ_{P_1} = 30 + 18.027 = 48.027 MPa

σ_{P_2} = 30 – 18.027 = 11.9722 ≈ 12 MPa

2.11 **(b)**

Given :

E = Young's modulus of elasticity

G = Shear modulus of elasticity

and ν = Poisson's ratio

We know that $E = 2G(1 + \nu)$ is relation between the three.

2.12 **(d)**

Given : Two bars

For bar 1 : L = Length

d = Diameter

P = Load

Let its Young's modulus = E

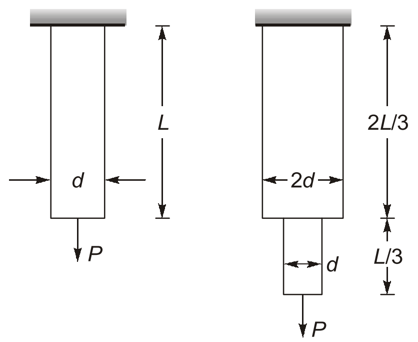
For bar 2 : L = Length

Dia = d for $\frac{L}{3}$

Dia = $2d$ for $\frac{2L}{3}$, Load = P

Let Young's modulus = E

To find : Ratio of strain energy stored in bar 1 to that of bar 2.



We know that strain energy stored

$$= \frac{1}{2} \cdot \frac{\sigma^2}{E} AL$$

where σ = Stress in bar

L = Length of bar

A = Area of bar

E = Young's modulus

For bar (1),

$$\text{Strain energy} = \frac{1}{2} \times \frac{P^2}{EA^2} \times AL = \frac{1}{2} \times \frac{P^2}{AE} \times L$$

$$= \frac{1}{2} \times \frac{4P^2}{\pi d^2} \times \frac{L}{E} \quad \text{...(1)}$$

Strain energy stored for bar (2) = Strain energy stored in $\frac{L}{3}$ length + strain energy stored in $\frac{2L}{3}$ length

$$= \frac{1}{2E}\frac{P^2}{A_1} \times \frac{L}{3} + \frac{1}{2E}\frac{P^2}{A_2} \times \frac{2L}{3}$$

$$= \frac{1}{2E}\frac{4P^2}{\pi d^2} \times \frac{L}{3} + \frac{1}{2E}\frac{4P^2 \times 2L}{\pi (2d)^2 \times 3}$$

$$= \frac{4P^2L}{2E\pi d^2 \times 3} + \frac{4P^2L \times 2}{2E\pi \times 4d^2 \times 3}$$

$$= \frac{6P^2L}{2E\pi \times d^2 \times 3} = \frac{2P^2L}{2\pi Ed^2} \quad ...(2)$$

Ratio = 1 : 2

$$= \frac{\text{Strain energy in bar 1}}{\text{Strain energy in bar 2}}$$

$$= \frac{4P^2L}{\pi d^2E} \times \frac{2\pi Ed^2}{2P^2L} = 2$$

2.15 (c)

Slender column is given and we have to find

$$\frac{(F_{cr}) \text{ both end clamped}}{(F_{cr}) \text{ both end hinged}}$$

F_{cr} = Critical buckling load

$$\frac{(F_{cr}) \text{ both end clamped}}{(F_{cr}) \text{ both end hinged}} = \frac{4\pi^2EI}{L^2} \times \frac{L^2}{\pi^2EI} = 4$$

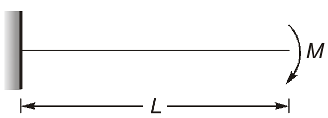
2.16 (a)

Given : Cantilever beam

E = Young's module, L = Length of beam

I = Moment of inertia, M = Moment at free end

To find : maximum deflection



We know that when a cantilever is subjected to moment M at free end, maximum deflection is $\frac{ML^2}{2EI}$.

2.17 (b)

Given : σ_{xx} = 40 MPa, σ_{yy} = 100 MPa

and τ_{xy} = 40 MPa.

To find the radius of the Mohr's circle representing the given state of stress in MPa is

We know that radius of Mohr's circle is

$$= \frac{1}{2}\sqrt{(\sigma_y - \sigma_x)^2 + 4\tau xy^2}$$

$$= \frac{1}{2}\sqrt{(100 - 40)^2 + 4 \times 40^2}$$

$$= \frac{1}{2}\sqrt{3600 + 6400} = 50 \text{ MPa}$$

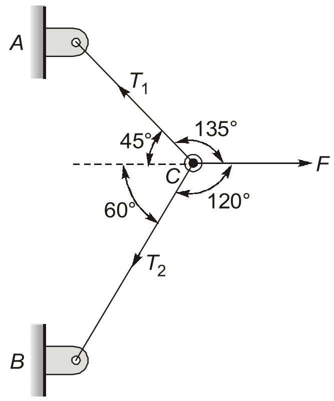
Radius of Mohr's circle for above stress is 50 MPa.

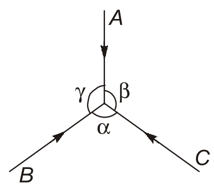
2.18 (b)

Given : Two truss members AC and BC with cross-sectional area of 100 mm^2 both. All joints are hinged.

To find : Force F that can be applied without exceeding a stress of 100 MPa.

Let T_1 = Tension in AC and T_2 is tension in BC.





Applying Lami's theorem,

$$\frac{T_1}{\sin 120°} = \frac{T_2}{\sin 135°} = \frac{F}{\sin 105°}$$

We know that according to Lami's theorem

$$\frac{A}{\sin\alpha} = \frac{B}{\sin\beta} = \frac{C}{\sin\gamma}$$

Here, we get, $T_1 = 0.8965\,F$ and $T_2 = 0.7320\,F$

So, maximum force is T_1. Stress generated in AC should not exceed 100 MPa, so

$$100 = \frac{T_1}{\text{Area}}$$

$$100 = \frac{0.8965F}{100}$$

$\Rightarrow$ F = 11154.48 N = 11.154 kN

2.19 (a)

To find : Vertical reaction at point B.

Vertical reaction at B be V_B.

$V_B = T_2 \sin 60°$

From previous question,

$T_2 = 0.7320\,F$

Here, F = 1 kN

So, T_2 = 0.7320 kN

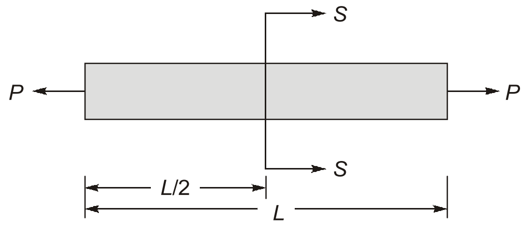
$V_B = 0.7320 \times \sin 60° = 0.63393$ kN

So, Vertical reaction at B is 0.63393 kN.

2.20 (a)

Given : L = Length of rod
A = Uniform cross sectional area

Young's modulus varies linearly from E_1 to E_2 along the length.



To find normal stress developed at section S-S

$$\text{Stress} = \frac{\text{Load}}{\text{Area}} = \frac{P}{A}$$

Stress does not depend on Young's modulus E.

2.22 (d)

Given: E = Young's modulus
G = Shear modulus
μ = Poisson's ratio

To find relationship between E, G and μ

We know that, $E = 2G(1 + \mu)$

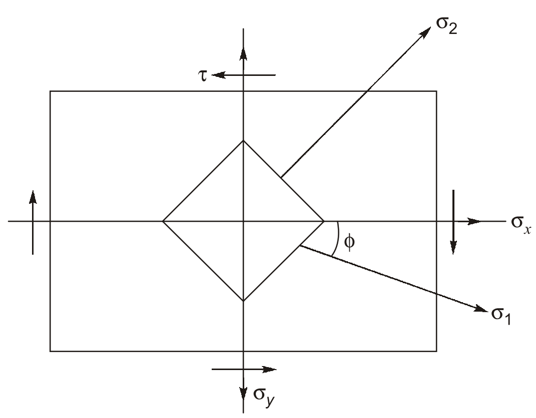
So, option (d) is correct.

2.23 (a)

Given : σ_x = Stress in x direction
= 5 kN/mm^2
σ_y = Stress in y direction
= 3 kN/mm^2
τ = Shear stress = 1 kN/mm^2

To find σ_1, σ_2, two principal stresses and direction ϕ of principal stress.

We know that principal stress is given by



$$\sigma_1, \sigma_2 = \frac{\sigma_x + \sigma_y}{2} \pm \frac{1}{2}\sqrt{(\sigma_y - \sigma_x)^2 + 4\tau x^2 y}$$

Here shear stress is –ve as it is acting in –ve direction of axes.

$$\sigma_1, \sigma_2 = \frac{5+3}{2} \pm \frac{1}{2}\sqrt{(3-5)^2 + 4\times(-1)^2}$$

$$= 4 \pm 1.4142$$

$$\sigma_1 = 4 + 1.4142 = 5.4142 \text{ kN/mm}^2$$

$$\sigma_2 = 4 - 1.4142 = 2.5858 \text{ kN/mm}^2$$

$$\tan 2\phi = \frac{2\tau}{\sigma_x - \sigma_y} = \frac{2(-1)}{5-3} = -\frac{2}{2} = -1$$

$$2\phi = -45^\circ$$

$$\phi = -22.5^\circ$$

So, $\sigma_1 = 5.4142$ kN/mm^2
$\sigma_2 = 2.5858$ kN/mm^2
$\phi = -22.5^\circ$

2.24 (c)

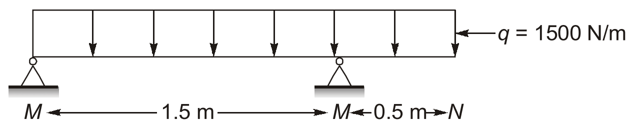
The bending movement is zero at point L and N only.

Given : Simply supported beam LMN with support at L and M. MN is overhanging part.

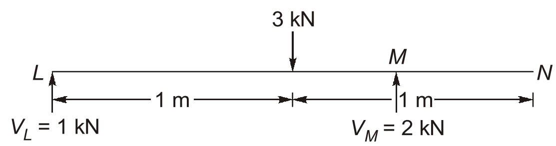
Uniformly distributed load,

$$q = 1500 \text{ N/m}$$

To find incorrect statement from given statements.



To find out reaction at L and M. Let us convert the UDL into point load.



UDL will act at centre and the magnitude of that point load will be 1500 × 2 N/m × m = 3000 N = 3 kN

Let reaction at L and M be V_L and V_M.

Total downward force = Upward force

$$V_L + V_M = 3000 \quad \text{...(1)}$$

Taking moment about L

$$3000 \times 1 = V_M \times 1.5$$

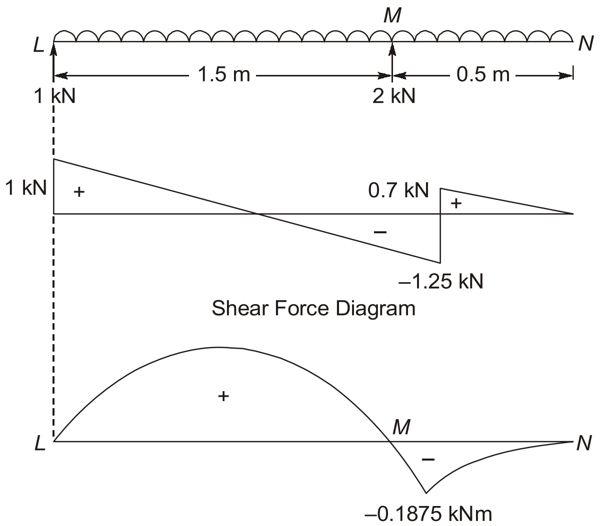
$$V_M = \frac{3000}{1.5} = 2000 \text{ N} = 2 \text{ kN}$$

$$V_L = 1000 \text{ N} = 1 \text{ kN}$$

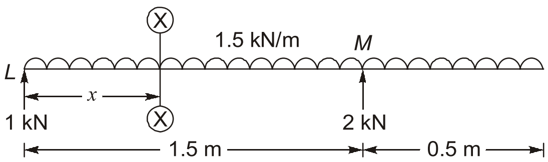
So, reaction at L and M are 1 kN and 2 kN.

Drawing shear force and bending moment.

Using section method given below :



Taking a section X-X at a distance x from L between L and M.



Shear force at section,

$$(SF)_X = 1 \text{ kN} - 1.5 \times x \text{ kN}$$

$x = 0$ at L, $(SF)_L = 1$ kN

$x = 1.5$ m at M,

$$(SF)_M = 1 - 2.25 = -1.25 \text{ kN}$$

Bending moment at section,

$$(BM)x = 1 \times x - 1.5(x)\frac{x}{2}$$

at $x = 0$ at L,

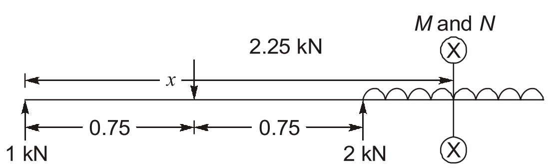
$$(BM)_L = 0 \text{ kNm}$$

at $x = 1.5$ m at M,

$$(BM)_M = 1 \times 1.5 - 1.5\frac{(1.5)^2}{2}$$

$$= -0.1875 \text{ kNm}$$

Now taking a section XX at distance x from L between M and N.



Converting UDL between L and M into point load which will act at 0.75 m from L and magnitude is 2.25 kN.

Shear force at section.

$$(SF)_X = 1 - 2.25 + 2 - 1.5(x - 1.5)$$

at $x = 1.5$ m, at M,

$$(SF)_M = 1 - 2.25 + 2 = 0.75 \text{ kN}$$

at $x = 2$ m, at N,

$$(SF)_N = 1 - 2.25 + 2 - 1.5(0.5) = 0$$

Bending moment at section,

$$(BM)_X = 1(x) - 2.25(x - 0.75) + 2(x - 1.5) - \frac{1.5(x - 1.5)^2}{2}$$

at $x = 1.5$ m, at M

$(BM)_M = 1.5 - 1.6875 + 0 - 0 = -0.1875$ kNM

at $x = 2$ m at N

$$(BM)_N = 2 - 2.8125 + 2(0.5) - \frac{1.5(0.5)^2}{2}$$

$= 2 - 2.8125 + 1 - 0.1875 = 0$ kNm

It is clear from the shear force and bending moment that bending moment is zero at ponts L, N and at a point between L and M.

Also, shear force is zero at points L and N and at a point between L and M.

So, incorrect option is (c). the bending moment is zero at pont L and N only.

2.25 (1.5811)

Given :

Density, $\delta = 3000$ kg/m^3

Diameter of marble,

$d = 0.025$ m

h = height from which marble is dropped

$h = 1$ km $= 1000$ m

Flow stress of roof material,

$\sigma = 2.5$ GPa $= 2.5 \times 10^9$ N/m^2

Acceleration due to gravity,

$g = 10$ m/s^2

To find depth indentation which is given by

$$\delta = 1000 \times \sqrt{\frac{\rho}{6\sigma}} \times d \times V \text{ mm}$$

When marble is dropped from 1 km, its potential energy is converted to kinetic energy

$$mgh = \frac{1}{2}mv^2$$

where m = mass and v = velocity

$$v = \sqrt{2gh} = \sqrt{2 \times 10 \times 1000}$$

$$= 141.421 \text{ m/s}$$

Depth of indentation (in mm),

$$\delta = 1000 \times \sqrt{\frac{3000}{6 \times 2.5 \times 10^9}} \times 0.025 \times 141.421$$

$= 1000 \times 4.4721 \times 10^{-4} \times 3.5355 = 1.5811$ mm

2.26 (414.201)

Given :

Initial diameter,

$$d_i = 13 \text{ mm}$$

Diameter at fracture,

$$d_f = 10 \text{ mm}$$

True stress at fracture = 700 MPa

To find engineering stress at fracture.

We know that

$$\text{True stress} = \frac{\text{Load}}{\text{Instantaneous area}}$$

$$\text{So, true stress at fracture} = \frac{\text{Fracture load}}{\text{Fracture area}}$$

$$\text{Also, Engineering stress} = \frac{\text{Load}}{\text{Initial area}}$$

Engineering stress at fracture

$$= \frac{\text{Fracture load}}{\text{Initial area}} \quad ...(2)$$

Using eqn. (1) and substituting values,

$$700 = \frac{\text{Fracture load}}{\frac{\pi}{4}(10)^2}$$

$$\Rightarrow \text{Fracture load} = \frac{700 \times \pi (10)^2}{4}$$

Putting this value of fracture load in eqn. (2)

$$\text{Engineering stress at fracture} = \frac{700 \times \frac{\pi}{4} \times (10)^2}{\frac{\pi}{4}(13)^2}$$

$$= \frac{700 \times 100}{13 \times 13} = 414.201 \text{ MPa}$$

So, engineering stress at fracture = 414.201 MPa.

2.27 (85)

Given : Principal stress values as

$$\sigma_{P1} = \sigma_{max} = 120 \text{ MPa}$$

$$\sigma_{P2} = \sigma_{min} = -50 \text{ MPa}$$

$$\sigma_{P3} = 10 \text{ MPa}$$

To find : maximum shear stress

We know that, maximum shear stress,

$$\tau_{max} = \frac{\sigma_{P\max} - \sigma_{P\min}}{2} = \frac{120 - (-50)}{2} = 85 \text{ MPa}$$

Maximum shear stress,

$$\tau_{max} = 85 \text{ MPa}$$

2.28 (2.5)

Given : specific weight of bar,

$$\gamma = 100 \text{ kN/m}^3$$

$$L = \text{Length of bar} = 100 \text{ m}$$

$$E = \text{Young's modulus} = 200 \text{ GPa}$$

To find elongation of bar due to its own weight.

We know that elongation of bar due to self weight is given as $\frac{\gamma L^2}{2E} = \delta l$, where δl = elongation.

On substituting values, we get

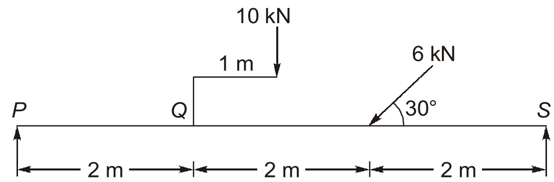
$$\delta l = 100 \times 1000 \text{ N/m}^3 \times \frac{(100)^2 \text{m}^2}{2 \times 200 \times 10^9} \frac{\text{m}^2}{\text{N}}$$

$$= \frac{100 \times 1000 \times (100)^2}{2 \times 200 \times 10^9} \text{ m}$$

$$= 2.5 \times 10^{-3} \text{ m}$$

$$\delta = 2.5 \text{ mm}$$

2.29 (14)



Given: Beam as shown in figure.

To find out : Bending moment at R.

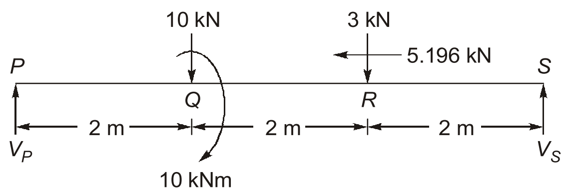
In the above beam, the frame attached at Q can be replaced by a load 10 kN at Q and a clockwise movement of 10 kNm at Q.

The load at R can be resolved into 2 components.

6 kN cos 30° horizontal = 5.196 kN horizontal

and 6 kN sin 30° vertical = 3 kN vertical

On doing as one changes the beam will be like as below :



Finding reactions at P and Q,

All upward forces = Downward forces

$$V_P + V_Q = 10 + 3$$

$$\Rightarrow \quad V_P + V_S = 13 \text{ kN} \quad ...(1)$$

Taking moment of all forces about P,

$$20 + 10 + 12 = V_S \times 6$$

$$42 \text{ kNm} = V_S \times 6 \text{ m}$$

$$V_S = 7 \text{ kN and } V_P = 6 \text{ kN}$$

If we consider right part of beam, i.e., RS, the bending moment at R due to reaction at S is $V_S \times$ 2m = 7 × 2 = 14 kNm. Bending moment at point R is 14 kNm.